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A Look at the Effects of Scattering on Polarimetry Using the NPDGamma Beam Monitors

Chad Gillis

This writeup addresses the problem of scattering in and around the ^{3}He polarizer of the NPDGamma experiment. Neutron and ^{3}He polarization calculations are made using beam monitor ratios to determine the transmission of the polarizer cell. This calculation is exact only if the transmission measurement that is made is of the ^{3}He inside the polarizer alone, or if the transmission from other materials cancels out of the ratio. There are many scattering materials located inbetween the beam monitors that are used to make this measurement, and it isn't immediately obvious that the scattering has a negligible effect. The conclusion reached by this writeup is that scattering affects both the neutron and ^{3}He polarization calculations by a multiplicative factor of less that one tenth of a percent.

What's needed to measure neutron and ^{3}He polarization

When initially unpolarized neutrons pass through a cell of polarized ^{3}He, they leave the cell with a polarization that can be determined if the ratio between the following two quantities is known:

- $T_{3,0}$ = the transmission of the ^{3}He when it's unpolarized; and
- $T_{3,P}$ = the transmission of the ^{3}He at the time for which we want to know the neutron polarization.

The neutron and ^{3}He polarizations can then be determined according to equation 1:

$$P_n = tanh(n_3 \sigma l P_3) = \sqrt{1 - \left(\frac{T_{3,0}}{T_{3,P}}\right)^2}. \tag{1}$$

P_n is the neutron polarization, n_3 is the ^{3}He volume density, σ is the well-known cross-section for capture of a neutron by ^{3}He, l is the width of the ^{3}He portion of the cell and P_3 is the ^{3}He polarization.

How is the measurement made and how would we calculate the polarization if we could ignore scattering effects?

Figure 1 represents the setup that was used for data-taking in March 2004.

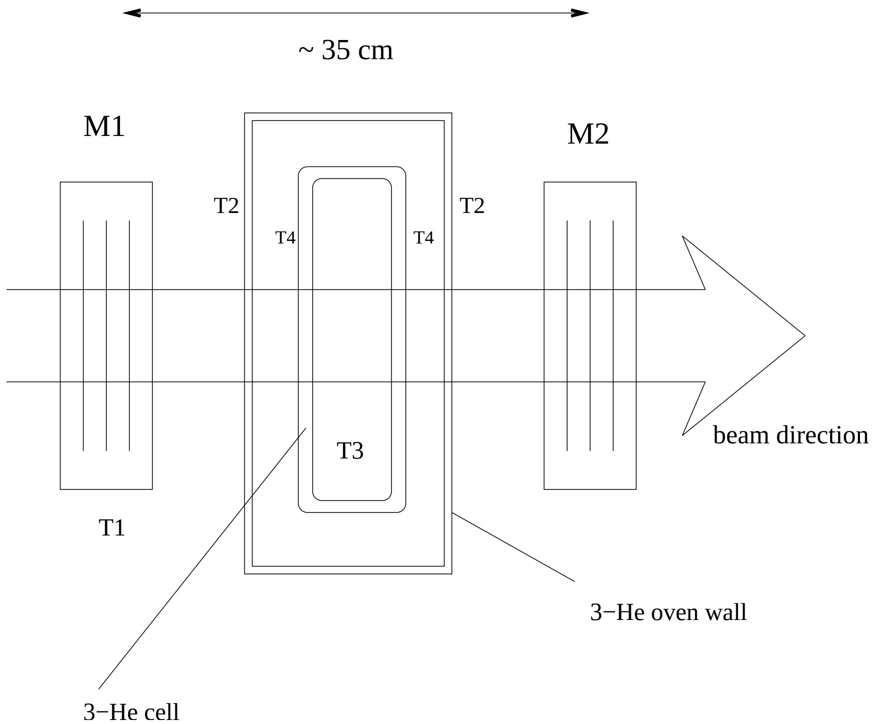

Figure 1: Monitors M1 and M2 are used to measure the neutron transmission of the polarizer cell. T_1 is the transmission of M1. T_3 is the transmission of the ^{3}He inside the polarizer, T_2 is the transmission of the silicon walls of the polarizer oven and T_4 is the transmission of the GE180 glass walls of the polarizer cell.

The goal is to determine the ratio between transmissions of the ^{3}He in the cell when it is unpolarized and when it is polarized. In the case of no interference from scattering, the problem is simple.

The properties of the monitors allow us to make the following statement: If the number of neutrons hitting a monitor is given as a function of energy by $N(E)$, then the current signal out of the monitor is given by $S(E) = K(E)N(E)$. For constant neutron energy the signal is proportional to the number of neutrons incident.

The signals S_1 in M1 and S_2 in M2 can then be expressed for an unpolarized cell as in equations 2 and 3.

$$S_{1,0} = K_1 N \tag{2}$$

$$S_{2,0} = K_2 T_1 (T_2 T_4)^2 T_{3,0} N \tag{3}$$

where N is the number of neutrons in the beam immediately before hitting M1.

As the cell is polarized, T_3 will change and N will in general also be different by the time we make the polarized measurement. S_1 and S_2 for a polarized cell are shown in equations 4 and 5.

$$S_{1,P} = K_1 N' \tag{4}$$

$$S_{2,P} = K_2 T_1 (T_2 T_4)^2 T_{3,P} N' \tag{5}$$

To determine the ratio of transmissions of unpolarized to polarized ^{3}He in the cell, it is sufficient to calculate the ratio of signals given by equation 6.

$$\frac{S_{2,0}/S_{1,0}}{S_{2,P}/S_{1,P}} = \frac{K_2 T_1 (T_2 T_4)^2 T_{3,0} N / K_1 N}{K_2 T_1 (T_2 T_4)^2 T_{3,P} N' / K_1 N'} = \frac{T_{3,0}}{T_{3,P}} \tag{6}$$

Statement of the problem and an estimation of the scattering involved

A complication presented by this measurement is that in addition to absorption from the ^{3}He, the monitors also see scattering from the material that exists between the two monitors. The scattering material consists of:

- Two 3-4 mm thick cell walls, made of Corning GE180 glass,
- Two 2 mm thick oven walls, made of pure Silicon, and
- 3.5 mm of Aluminum inside the monitors themselves.

Ideally M1 and M2 would occupy as small of a solid angle as possible relative to any source of scattering so that a scattered neutron could be approximated as an absorbed neutron. Any effects from scattering would then cancel out when the ratio is taken of the transmissions of the cell polarized and unpolarized. However, for data-taking in March/04, the cell was placed inbetween the two monitors as shown in figure 1, so it is not immediately obvious whether the scattering affected the polarization calculation in a significant way.

Another issue is that the polarization of a neutron may be altered when it is scattered. However, this writeup does not address this issue.

The transmission of GE180 glass was measured using the beam monitors to vary (over the energies 4 meV to 50 meV) between 0.92 and 0.93 for a 3.5 mm thick window.

The transmission of pure silicon can be estimated. From ENDF data, the scattering cross-section for neutrons on natural silicon is about 2 barns. The density of Silicon is 2.33

g/cc and the atomic mass is 28.0855. The transmission of 2 mm of Silicon should therefore be about $T = exp(-n\sigma l) = exp(-N_A * \frac{2.33g}{28.0855g/cm^3} * 2barns * 1mm) = 0.94$.

Aluminum scatters about 1% per mm so the probability for transmission through the aluminum should be about $T_1' = 0.97$. The transmission of the monitors is dependent on energy due to the ^{3}He that they contain, but for the purposes of this discussion, the total transmission of the monitors will be taken to be $T_1 = 0.95$, independent of energy. Here T_1 is the probability for transmission through the whole monitor and T_1' is the probability for not getting scattered out of the monitor.

To summarize the results:

$$T_1 \approx 0.95 \qquad T_1' \approx 0.97 \qquad T_2 \approx 0.94 \qquad T_4 \approx 0.91$$

An estimation of the solid angles

Now that we know how much scattering occurs, the next step is to figure out what solid angles the monitors subtend about the scattering materials. The solid angle contribution from each piece of scattering material will be considered separately. The solid angles to be considered are shown in figure 2.

In order to calculate the effective solid angle for a given window, it would be possible to integrate the contributions from infinitesimal elements all over the whole window. However, the element at the center of a slab of scattering material will contribute the largest amount to the effective solid angle. So if we treat the whole window as if it corresponded to the same solid angle as the center, then we will end up with an overestimate of the contribution from scattering. If the effect determined in this manner is negligible, then the real effect will also be negligible.

The distances are as follows (the position of each monitor is taken from its center):

M1 to first silicon window	:	5 cm
M1 to first GE180 window	:	13.8 cm
M1 to second GE180 window	:	19 cm
M1 to second silicon window	:	28 cm
M2 to first silicon window	:	31 cm
M2 to first GE180 window	:	22.4 cm
M2 to second GE180 window	:	17 cm
M2 to second silicon window	:	8.3 cm

M1 to M2 : 35.5 cm

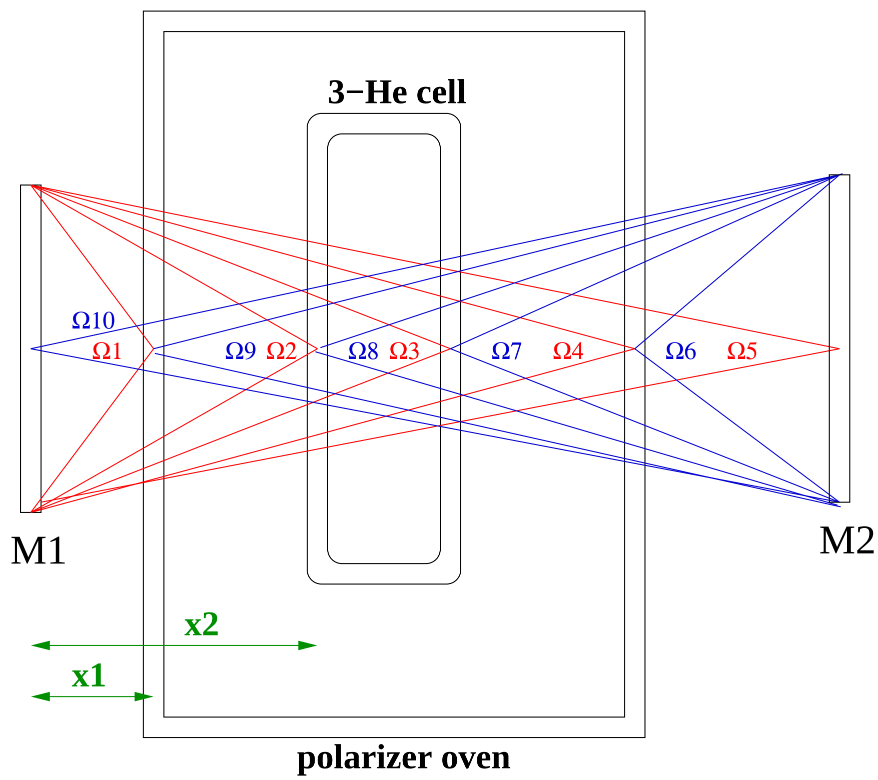

Figure 2: Each solid angle to be considered is shown labeled in this diagram. x_i are the distances which correspond to the solid angles Ω_i.

The face of each monitor is 12cm x 12 cm.

Each solid angle can then be estimated using equation 7:

$$\frac{\Omega_i}{4\pi} = \frac{1}{2}\left(1 - \frac{x_i}{\sqrt{x_i^2 + (a/2)^2}}\right) \tag{7}$$

where a is something a bit larger than 12 cm. Using this method, the following values were found:

$$\tfrac{\Omega_1}{4\pi} = 0.20 \qquad \tfrac{\Omega_2}{4\pi} = 0.051 \qquad \tfrac{\Omega_3}{4\pi} = 0.029 \qquad \tfrac{\Omega_4}{4\pi} = 0.014 \qquad \tfrac{\Omega_5}{4\pi} = 0.0089$$

Ω_6 to Ω_{10} will cancel out of the final answer so they can be ignored.

Determination of the size of the effect from scattering

In order to decide what the size of the scattering effect is, only first-order corrections will be considered. Here a first-order scattering correction is one which treats the second scatter of any neutron to be equivalent to an absorption. Also, any scatter will be considered to be isotropic in the lab.

The observed signal in M1 will therefore be the number of neutrons in the beam before hitting M1, plus first-order corrections due to backscatter and subsequent losses on their way back to M1. Referring to figures 1 and 2, the expression for S_1 becomes:

$$\begin{aligned} S_1 = K_1N & \\ &+ K_1NT_1(1-T_2)\frac{\Omega_1}{4\pi} \\ &+ K_1NT_1T_2(1-T_4)\frac{\Omega_2}{4\pi}T_2 \\ &+ K_1NT_1T_2T_4T_3(1-T_4)\frac{\Omega_3}{4\pi}T_3T_4T_2 \\ &+ K_1NT_1T_2T_4T_3T_4(1-T_2)\frac{\Omega_4}{4\pi}T_4T_3T_4T_2 \\ &+ K_1NT_1T_2T_4T_3T_4T_2(1-T_1')\frac{\Omega_5}{4\pi}T_2T_4T_3T_4T_2 \\ &= K_1N(A + BT_3^2) \end{aligned} \tag{8}$$

where A = 1.015 and B = 0.0024.

Similarly, the signal in M2 (S_2) is the number of neutrons that make it unaffected to M2, plus first order corrections due to forward scatter and subsequent losses on their way forwards to M2:

$$S_2 = K_2NT_1T_2T_4T_3T_4T_2$$

$$
\begin{aligned}
&+ K_2NT_1T_2T_4T_3T_4(1-T_2)\frac{\Omega_6}{4\pi} \\
&+ K_2NT_1T_2T_4T_3(1-T_4)T_2\frac{\Omega_7}{4\pi} \\
&+ K_2NT_1T_2(1-T_4)T_3T_4T_2\frac{\Omega_8}{4\pi} \\
&+ K_2NT_1(1-T_2)T_4T_3T_4T_2\frac{\Omega_9}{4\pi} \\
&+ K_2N(1-T_1')T_2T_4T_3T_4T_2\frac{\Omega_{10}}{4\pi} \\
&= K_2NxT_3 \qquad (9)
\end{aligned}
$$

where x is irrelevant as it will cancel out before the final result is reached.

The ratio between M1 and M2 for a given data-taking run is then given by equation 10.

$$R = \frac{K_2NxT_3}{K_1N(A+BT_3^2)} \qquad (10)$$

So the ratio of M2/M1 (unpolarized) to M2/M1 (polarized) is given by equation 11.

$$\frac{R_0}{R_P} = \frac{K_2NxT_{3,0}/K1N(A+BT_{3,0})}{K_2N'xT_{3,P}/K1N'(A+BT_{3,P})} = \left[\frac{T_{3,0}}{T_{3,P}}\right]\left\{\frac{A+BT_{3,P}}{A+BT_{3,0}}\right\} \qquad (11)$$

The factor that's in the square brackets is the required result while the quantity on the left-hand side is what we measure. The factor in the curly brackets is therefore a first-order correction to what we measure which accounts for the fact that scattering did indeed take place. Since $T_{3,P} > T_{3,0}$, it follows that the correction factor will be greater than one. If the correction were not taken into account, calculated polarization would therefore be less than actual polarization as expressed by equation 12.

$$\frac{R_0}{R_P} > \frac{T_{3,0}}{T_{3,P}} \iff \sqrt{1-\left(\frac{R_0}{R_P}\right)^2} < \sqrt{1-\left(\frac{T_{3,0}}{T_{3,P}}\right)^2} \qquad (12)$$

The goal is to find out to what extent this correction is different from one. An extreme example would be for an infinitely thick cell which could be polarized to 100%. Then $T_{3,0} = 0$ and $T_{3,P} = 0.5$. In that case the correction factor deviates from unity by 0.015%.

How big of a correction does this end up being for the conditions of our experiment?

In order to determine the answer to this question, the following assumptions were made.

- The polarization of the ^{3}He in the cell $P_3 = 46$ %. This is based on calculations that were made using beam monitor transmission measurements.

- The amount of ^{3}He in the cell as seen by M2 is $n\sigma$l = 0.0714 ms^{-1}tof. This is the ^{3}He aerial density times the neutron-^{3}He cross section, which is directly proportional to neutron time of flight. This quantity was determined using beam monitor transmission measurements.

Using these assumptions and the information presented so far in this writeup, the following calculations were made:

- $T_{3,0} = e^{-n\sigma l}$
- $T_{3,P} = T_{3,0}\ cosh(n\sigma l P_3)$
- $P_n = \sqrt{1 - \left(\frac{T_{3,0}}{T_{3,P}}\right)^2}$
- $R_0/R_P = \frac{T_0}{T}\frac{A+BT}{A+BT_0}$
- $P_n' = \sqrt{1 - \left(\frac{R_0}{R_P}\right)^2}$
- $P_3' = arctanh(P_n')/n\sigma l$

The results are shown in the following table:

tof (ms)	P_n	P_n'	P_n'/P_n	P_3'	P_3'/P_3
2	6.56	6.55	0.9990	45.95	0.9990
4	13.06	13.05	0.9991	45.96	0.9991
6	19.46	19.44	0.9992	45.96	0.9992
8	25.69	25.67	0.9993	45.97	0.9993
10	31.71	31.69	0.9994	45.97	0.9994
12	37.49	37.47	0.9995	45.98	0.9995
14	42.99	42.98	0.9996	45.98	0.9995
16	48.19	48.18	0.9996	45.98	0.9996
18	53.08	53.06	0.9997	45.98	0.9996
20	57.63	57.61	0.9997	45.98	0.9997
22	61.85	61.84	0.9998	45.99	0.9997
24	65.74	65.73	0.9998	45.99	0.9997
26	69.31	69.30	0.9998	45.99	0.9998
28	72.57	72.56	0.9999	45.99	0.9998
30	75.54	75.53	0.9999	45.99	0.9998
32	78.22	78.21	0.9999	45.99	0.9998
34	80.64	80.64	0.9999	45.99	0.9998
36	82.82	82.81	0.9999	45.99	0.9999
38	84.77	84.77	0.9999	45.99	0.9999
40	86.52	86.52	1.0000	45.99	0.9999

Conclusion

For the conditions that existed during the NPDGamma commissioning run in the winter and spring of 2004, it is acceptable to use monitor transmission ratios in the calculation of ^{3}He and neutron polarizations. According to the assumptions and calculations made in this report, corrections due to scattering around the polarizer are a multiplicative factor of a tenth of a percent or less.